Seminar 3

Here is the translated Quarto document.

Problem 1

When passing one rapid, a kayak sustains no damage with probability p_1, receives serious damage with probability p_2, and breaks completely with probability $p_3 = 1 - p_1 - p_2$. Two serious damages lead to a complete breakage. Find the probability that after passing n rapids, the kayak will not be completely broken.

Problem 2

Let $n \geq 2$. A number is chosen randomly from $1, 2, \ldots, n$. Event A is that the chosen number is divisible by 2, and event B is that the chosen number is divisible by 7. Find all n such that events A and B are independent.

Problem 3

(Geometric distribution) Two players take turns rolling a die. The first one to roll a 6 loses.

a. Find the probability that a round consists of exactly n rolls.
b. Find the probability that the first player loses.

Problem 4

a. Let event A be independent of itself. What is its probability?
b. Let $\mathbb{P}(A) = 0$ or $\mathbb{P}(A) = 1$. Show that event A is independent of any event B.

Problem 5

A die is rolled until a number less than five is obtained for the first time. What is the probability that the last roll is at least two?

Problem 6

Alice and Bob play the following game. A fair coin is tossed until the sequence 110 or 100 appears. Alice wins if 110 appears first, and Bob wins if 100 appears first. Who will win more often? What are the probabilities of Alice's and Bob's wins?

Problem 7

Let p_n denote the probability that in n tosses of a fair coin, three consecutive heads do not appear. Find a recurrence relation for $ p_n $.

Problem 8

Events A, B, and C are pairwise independent and have equal probabilities. Given that $A \cap B \cap C = \emptyset$, find the maximum possible value of $\mathbb{P}(A)$.

Problem 9

According to the schedule, a bus and a trolleybus run every 20 minutes until midnight. The trolleybus starts at 6:00, and the bus at 6:15. Find the probability of leaving by trolleybus, if you arrive at the stop at a random time during the day and take the first vehicle that arrives.

Problem 10

X and Y agreed to meet between 12:00 and 13:00. Each is willing to wait for exactly 30 minutes. What is the probability that they meet? What is the probability that they meet and X did not wait for Y? What is the probability that they arrive at the same time?

Problem 11

A standard computer generator `rand` produces random numbers on the interval from zero to one. Then, the square root of each number is taken, and the answer is printed in a fixed-point format with 16 digits of precision after the decimal point (e.g., like this: $0.0003267891135015\ldots$). Find the probability that in this notation, the second digit after the decimal point will be a two. Find the answer analytically and compare it with the result of a computer experiment.

Problem 12

Three people each choose a number from the interval $$. What is the probability that a triangle with these side lengths exists?